Exercise sheet #7

Problem 1. A very long cylinder, of radius a, carries a uniform polarization $\mathbf{P}$ perpendicular to its axis. Find the electric field inside the cylinder. Show that the field outside the cylinder can be expressed in the form

$$\mathbf{E}(\mathbf{r}) = \frac{a^2}{2\epsilon_0 s^2}[2(\mathbf{P}\cdot\hat{\mathbf{s}})\hat{\mathbf{s}} - \mathbf{P}]$$

[Careful: I said "uniform," not "radial"!]

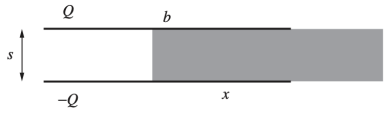
Problem 2. A short cylinder, of radius a and length L, carries a "frozen-in" uniform polarization $\mathbf{P}$, parallel to its axis. Find the bound charge, and sketch the electric field (i) for $L \gg a$, (ii) for $L \ll a$, and (iii) for $L \approx a$. [This is known as a bar electret; it is the electrical analog to a bar magnet. In practice, only very special materials-barium titanate is the most "familiar" example-will hold a permanent electric polarization. That's why you can't buy electrets at the toy store.]

Problem 3. For the bar electret of the last problem, make three careful sketches: one of $\mathbf{P}$, one of $\mathbf{E}$, and one of $\mathbf{D}$. Assume L is about $2a$. [Hint: $\mathbf{E}$ lines terminate on charges; $\mathbf{D}$ lines terminate on free charges.]

Problem 4. A rectangular capacitor with side lengths a and b has separation s, with s much smaller than a and b. It is partially filled with a dielectric with dielectric constant κ. The overlap distance is x; see the figure below. The capacitor is isolated and has constant charge Q.

(a) What is the energy stored in the system? (Treat the capacitor like two capacitors in parallel.)

(b) What is the force on the dielectric? Does this force pull the dielectric into the capacitor or push it out?



Problem 5. A metal sphere of radius a carries a charge Q (See figure below). It is surrounded, out to radius b, by linear dielectric material of permittivity ϵ. Find the potential at the center (relative to infinity).